LSC 5.5 Hard Mathematics: Effective Derivations, Consistency Conditions, and Scale Estimates

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Abstract

This technical companion strengthens the mathematical core of the final `LSC 5.5` framework. The aim is not to claim a complete first-principles theory, but to push the model into a cleaner phenomenological form with explicit dimensional checks, effective derivations, small-parameter expansions, and scale estimates. The document develops: an effective Schr"odinger evolution equation in flavor space; a resonance criterion generated by a PBH-modulated geometric term; a tensor reconstruction model

for observed energy; a linearized event-rate shift formula; a covariant effective action; and minimal consistency conditions for the ansatz. The resulting structure is suitable for preprint-level presentation and for later numerical fitting.

1. Effective flavor-space dynamics

Let the flavor vector be

$$|\nu(x)\rangle = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix},$$

and let its evolution along an affine parameter `\lambda` obey

$$i\frac{d}{d\lambda}|\nu(\lambda)\rangle = H_{\text{eff}}(\lambda)|\nu(\lambda)\rangle.$$

We write

$$H_{\text{eff}} = \frac{1}{2E}UM^2U^\dagger + V_m + V_g + V_{\text{LSC}},$$

with

$$V_m = \text{diag}(V_e, 0, 0), \qquad V_e = \sqrt{2}G_F n_e(r),$$

$$V_g = \beta_g \frac{E}{c^2}\Phi(r)\,\mathbb{I}, \qquad \Phi(r) = -\frac{GM}{r},$$

$$V_{\text{LSC}} = \alpha_{\text{LSC}}\left(\frac{GM}{rc^2}\right)F(E)\,\Xi.$$

Here `\beta_g` is an explicit bookkeeping factor. A strictly flavor-universal term proportional to the identity does not by itself change oscillation probabilities, but it is retained because the model treats the total effective phase and the detector sector as coupled at the phenomenological level.

2. Two-flavor reduction and resonance

For analytical control, reduce to a two-state system with vacuum parameters `(\theta,\Delta m^2)`. The effective Hamiltonian may then be written as

$$H_2 = \frac{\Delta m^2}{4E}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \frac{1}{2}\begin{pmatrix} V & 0 \\ 0 & -V \end{pmatrix},$$

where

$$V = V_{\text{matter}} + V_{\text{LSC}}.$$

Diagonalization gives the standard effective quantities

$$\tan 2\theta_{\text{eff}} = \frac{\Delta m^2 \sin 2\theta}{\Delta} m^2 \cos 2\theta - 2EV,$$

$$\Delta m^2_{\text{eff}} = \sqrt{(\Delta m^2 \cos 2\theta - 2EV)^2 + (\Delta m^2 \sin 2\theta)^2}.$$

Resonance occurs when the diagonal splitting vanishes:

$$\Delta m^2 \cos 2\theta = 2EV.$$

Substituting the LSC piece yields

$$\Delta m^2 \cos 2\theta = 2E \left[\sqrt{2} G_F n_e(r) + \alpha_{\text{LSC}} \left(\frac{GM}{rc^2} \right) F(E) \right].$$

This equation is one of the core hard-math statements of the model because it turns the resonance branch into an explicit parameter constraint.

3. Choice of the flavor operator `\Xi`

The earlier documents left `\Xi` mostly implicit. To make the model sharper, a minimal phenomenological choice is

$$\Xi = \begin{pmatrix} 1 & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon^*_{e\mu} & 0 & \epsilon_{\mu\tau} \\ \epsilon^*_{e\tau} & \epsilon^*_{\mu\tau} & -1 \end{pmatrix},$$

with Hermiticity enforced by complex conjugation in the off-diagonal entries. This form permits:

- diagonal gravitational flavor bias,
- off-diagonal geometric mixing,
- a smooth decoupling to standard oscillations when all `\epsilon` terms and `\alpha_{\mathrm{LSC}}` vanish.

For a one-parameter minimal model, set the off-diagonal terms to zero and keep only the diagonal splitting. For a richer fit, allow one off-diagonal entry at a time.

4. Tensor energy reconstruction

The detector sector is promoted from a scalar shift to a tensor structure:

$$\Delta_D = D_{\mu\nu} p^\mu p^\nu.$$

Because `p^\mu p^\nu` has dimensions of energy squared in natural units, `D_{\mu\nu}` must carry dimensions of inverse energy squared if `\Delta_D` is to be dimensionless. This is an important consistency condition.

Write

$$E_{\text{true}} = E_{\text{obs}}\left(1 + \alpha_D \Delta_D\right),$$

where `\alpha_D` is dimensionless and `\Delta_D` is dimensionless.

For a static detector environment with a preferred spatial direction `\hat{n}`, one convenient decomposition is

$$D_{\mu\nu} = d_t\, u_\mu u_\nu + d_s\, \Pi_{\mu\nu} + d_a\, n_\mu n_\nu,$$

where:

- `u_\mu` is the detector four-velocity,
- `\Pi_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu` projects onto the local rest-space,
- `n_\mu` is a unit spacelike anisotropy vector,
- `d_t`, `d_s`, `d_a` have dimensions of inverse energy squared.

For a relativistic neutrino with `p^\mu \simeq E(1,\hat{p})`, this gives

$$\Delta_D \simeq E^2 \left[d_t + d_s + d_a (\hat{n}\cdot\hat{p})^2\right],$$

so that the anisotropy enters through a quadrupole-like directional factor.

5. Scalar limit and small-bias expansion

If

$$D_{\mu\nu} = d_0\, \eta_{\mu\nu},$$

then for an exactly massless particle `p^\mu p_\mu = 0` and the correction would vanish. This shows that the scalar `\delta` model cannot literally be the same as a pure Minkowski-proportional tensor model. The correct statement is weaker: the earlier scalar model should be viewed as an effective low-resolution limit of the tensor formalism after averaging over detector structure and reconstruction algorithms.

Thus, instead of imposing exact equality, define the scalar coarse-grained limit by

$$\langle \alpha_D D_{\mu\nu} p^\mu p^\nu \rangle_{\text{det}} \equiv \delta_{\text{eff}}(E,\Omega).$$

This is mathematically cleaner than claiming strict identity with `d_0 \eta_{\mu\nu}`.

6. Event-rate perturbation theory

Let

$$N = \int dE\, \mathcal{I}(E), \qquad \mathcal{I}(E) = \Phi(E) P(E) \sigma(E) R(E).$$

Assume a small reconstruction bias

$$E \to E(1+\varepsilon), \qquad |\varepsilon| \ll 1.$$

Then

$$\mathcal{I}(E(1+\varepsilon)) \approx \mathcal{I}(E) \left[\frac{d\ln \mathcal{I}}{d\ln E}\right] \varepsilon\, \mathcal{I}(E),$$

which implies

$$\frac{\Delta N}{N} \approx \left\langle \frac{d\ln \Phi}{d\ln E} + \frac{d\ln P}{d\ln E} + \frac{d\ln \sigma}{d\ln E} + \frac{d\ln R}{d\ln E} \right\rangle \varepsilon.$$

If `\sigma(E)\propto E^2`, then one contribution is fixed:

$$\frac{d\ln \sigma}{d\ln E} = 2.$$

This result is central because it shows that the event-rate anomaly is not set by the energy bias alone, but by the logarithmic sensitivity of the entire integrand.

7. Order-of-magnitude estimate

Take a coarse low-energy estimate in which the weighted logarithmic slope is of order

$$\mathcal{S} = \left\langle \frac{d\ln \Phi}{d\ln E} + \frac{d\ln P}{d\ln E} + \frac{d\ln \sigma}{d\ln E} + \frac{d\ln R}{d\ln E} \right\rangle \sim 2 \text{ to } 4.$$

Then

$$\frac{\Delta N}{N} \sim \mathcal{S}\, \varepsilon.$$

To generate a `10\%` rate distortion, one needs roughly

$$\varepsilon \sim \frac{0.1}{\mathcal{S}} \sim 0.025 \text{ to } 0.05.$$

This means a few-percent reconstruction bias is already in the right ballpark for a phenomenologically visible effect. That does not prove the model, but it does show that it is not trivially scale-forbidden.

8. Geometric sourcing of the detector tensor

A cleaner effective ansatz than the earlier loose forms is

$$D_{\mu\nu} = \frac{1}{\Lambda_D^2} \left[a\, g_{\mu\nu} + b\, \frac{R_{\mu\nu}}{\mathcal{R}_0} + c\, u_\mu u_\nu + d\, n_\mu n_\nu \right],$$

where:

- `\Lambda_D` is the detector reconstruction scale,
- `\mathcal{R}_0` is a reference curvature scale introduced only to make the coefficient multiplying `R_{\mu\nu}` dimensionless,
- `a`, `b`, `c`, `d` are dimensionless effective parameters.

Now the dimensions are explicit:

- `D_{\mu\nu}` has dimensions `[\mathrm{energy}]^{-2}`,
- `D_{\mu\nu}p^\mu p^\nu` is dimensionless,
- the tensor decouples as `\Lambda_D \to \infty`.

9. Covariant effective action and field equation

Take the effective action

$$S = \int d^4x \sqrt{-g} \left[i\bar{\psi}\gamma^\mu \nabla_\mu \psi - m\bar{\psi}\psi - G_D \bar{\psi}\gamma^\mu D_{\mu\nu}\gamma^\nu \psi \right].$$

Variation with respect to `\bar{\psi}` yields

$$(i\gamma^\mu \nabla_\mu - m - G_D \gamma^\mu D_{\mu\nu}\gamma^\nu)\, \psi = 0.$$

If `D_{\mu\nu}` is treated as an external background tensor, this is enough for a consistent effective field equation. If one wants `D_{\mu\nu}` to become dynamical, an additional kinetic and potential sector must be supplied, which the present framework does not yet do. This is another important boundary of the current model.

10. Hermiticity and positivity constraints

For a viable effective Hamiltonian:

1. `H_{\mathrm{eff}}` must be Hermitian in flavor space,
2. `D_{\mu\nu}` should be real and symmetric at the effective level,
3. the reconstructed energy should remain positive:

$$1 + \alpha_D \Delta_D > 0.$$

If the correction is perturbative, it is sufficient to require

$$|\alpha_D \Delta_D| \ll 1.$$

This condition also justifies the linear event-rate expansion.

11. Minimal parameter set for fitting

A reduced practical parameter set for first scans is:

$$\{\alpha_{\mathrm{LSC}}, \Lambda_D, d_a, \epsilon_{e\mu}, M, r\}.$$

Interpretation:

- `\alpha_{\mathrm{LSC}}` controls the size of the propagation modification,
- `\Lambda_D` controls the detector-sector suppression scale,
- `d_a` controls anisotropy strength,
- `\epsilon_{e\mu}` controls minimal off-diagonal flavor coupling,
- `M` and `r` determine the local geometric environment.

Everything else can be frozen to standard oscillation values in a first pass.

12. What is genuinely improved here

Compared with the softer theory document, the hard-math version does four useful things:

1. it gives an explicit two-flavor resonance derivation,
2. it fixes the dimensions of `D_{\mu\nu}`,
3. it replaces the misleading exact scalar-tensor identification with a coarse-grained statement,
4. it supplies a concrete scale estimate showing that a few-percent energy bias can plausibly map into a ten-percent event-rate effect.

13. What still remains open

The following pieces remain open:

- a microphysical derivation of `H_{\mathrm{LSC}}`,
- a detector-theory derivation of `D_{\mu\nu}`,
- a full three-flavor numerical scan,
- explicit experimental likelihoods,
- astrophysical consistency bounds on the PBH environment.

14. Conclusion

The `LSC 5.5` framework can be cast into a significantly stronger mathematical form without pretending to have solved the full theory from first principles. The hard-math upgrade supplied here makes the effective structure cleaner, dimensionally controlled, and better suited for preprint use. In particular, it shows how the model can be written as a coupled propagation-plus-reconstruction phenomenology with explicit resonance conditions, tensor reconstruction, and order-of-magnitude event-rate amplification.